

# Equipment Selection Models for Space Systems: Cost-Effectiveness, Reliability, and Maintainability

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Several models are developed to illustrate a methodology for equipment selection. By equipment selection we mean assigning computers to applications so that cost, effectiveness, reliability, maintainability, and availability requirements are satisfied. Because so many facets of the assignment problem must be addressed, the models must be developed and applied incrementally as each requirement is assessed. For example, if equipment had been found desirable from a cost-effectiveness standpoint, it might not hold up under the scrutiny of reliability and maintainability assessments. As a consequence, equipment assignments to applications would have to be revised. To illustrate the assignment process we use the organizational scenario of a space system laboratory that operates an integrated system of web service, safety critical, and home computer applications.

## I. Introduction to Equipment Selection

THE selection of a computer is dependent on many criteria. Total cost-effectiveness is paramount, not only initial hardware costs and effectiveness but effectiveness over a period of several years. Not to be ignored is the cost of maintenance and the reliability that contributes to maintainability. Organizations usually have a requirement to select a set of equipment to satisfy various application requirements [1]. To state this idea formally, the organization chooses from among  $M$  equipment types (i.e., models) to satisfy  $N$  applications. In doing so, the objective is to make the choice such that total cost-effectiveness over the time horizon is maximized. The organizational environment we choose to illustrate our equipment selection models is a space systems laboratory that operates three applications: web service, safety critical, and home computer. These are coordinated applications. That is, the web service application provides access to space flight data to support space missions, the safety critical application is a series of planetary missions, and the home computer application provides laboratory scientists with computing capability to support space flights from home.

The execution of the three applications is scheduled so that web services provide support to mission control during space flights and to the home computer application. Although some of the application executions are overlapped, we assume they are disjoint for the purpose of using a conservative time availability constraint.

The space laboratory maintains a nominal inventory of equipment types. By nominal we mean according to anticipated usage and expected reliability, maintainability, and availability. The laboratory does not stock inventory at maximum levels. Therefore, it is possible to incur a shortage.

Selection of spaceborne computing platforms requires balance among several competing factors. Traditional performance analysis techniques are ill-suited for this purpose due to their overriding concern with runtime [2].

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We agree that there should be balance among competing factors. In addition, there are macro level concerns that we address, such as cost, reliability, and maintainability that must be addressed in space system applications.

### A. Model Definitions

Definitions important to the development of the selection models follow [3].

#### *Equipment Factors*

Equipment type	synonymous with equipment model
Equipment factor	property of equipment type (e.g., clock speed)
Equipment effectiveness factor	value of an equipment factor (e.g., <i>value</i> of clock speed)
$E$	equipment effectiveness metric
$f_{ki}$	equipment effectiveness factor $k$ for equipment type $i$
$f_{kj}$	equipment effectiveness factor required by application $j$
$K$	number of equipment effectiveness factors
$M_i$	number of equipments of type $i$
$X_{ij}$	number of equipment types $i$ assigned to application $j$

#### *Cost Factors*

$C_i$	fixed cost per of equipment type $i$
$C_v$	total variable cost
$c_{ij}$	variable operating cost per unit time for using equipment $i$ on application $j$
$r$	alternate rate of return (rate of return obtainable on alternate investment) [4]

For example an Internet service provider could decide to invest in a new search engine rather than web services. A public transportation authority could decide on improving highways rather than funding a light rail system. A home owner could decide to buy a new automobile rather that purchase a new computer.

$EC(i, k)$	effectiveness-cost (EC) ratio of equipment type $i$ with respect to factor $k$
$EC(i, j, k)$	EC ratio of assigning equipment type $i$ to application $j$ with equipment factor $k$
$PV(i, j)$	present value (PV) of assigning equipment type $i$ to application $j$ : result of discounting future costs by alternate rate of return

#### *Application Factors*

$N$	number of applications
$n$	number of years of planned operation for each application
$t_j$	scheduled application $j$ computer execution time per session or mission
$T_j$	time that is available to be used on application $j$

#### *Reliability Factors*

$f_r$	mean failure rate
$R(t_j)$	single computer reliability of application $j$
$R_p(t_j)$	parallel computer reliability of application $j$
$TU_j$	time that application $j$ is inoperable due to unreliable operation
$\lambda_1$	initial failure rate
$\lambda(t_j)$	failure rate of application $j$

#### *Maintainability Factors*

$M(TM_j)$	maintainability of application $j$ (probability that repair will require time $TM_j$ )
$TM_j$	time that application is down due to maintenance

## B. Process Flow Diagram of Equipment Assignment Model

Selection of a computer for an engineering environment can be a terrifying and frustrating experience for the individual who is unfamiliar with the computer and just exactly what it can do. The situation is complicated by the vast number of alternatives available to the prospective buyer of computer equipment. The frustration and fear can be alleviated and dealt with by designing a procedure that is sound and that will prevent the neophyte from making any serious mistakes if that procedure is adhered to carefully. We design such a procedure, as illustrated in Fig. 1 [5]. The process flow diagram of equipment assignment model that includes cost, effectiveness, reliability, maintainability, and availability is shown. The purpose of this diagram is to provide an overview of the model and how the components interact.

## II. Equipment Assignment Cost Model

Compute total variable cost to use in the PV equation (2):

$$C_v = c_{ij} * t_j \quad [3] \quad (1)$$

An approach to optimize the assignment of equipment is to minimize the time discounted value of the organization's computing costs while still meeting service constraints (equipment and time availability). Therefore, we minimize *expected* PV of investment in equipment  $i$  assigned to application  $j$  for time  $t_j$ , discounting future costs with the rate of return on alternate investments  $r$  [6]. It is not sufficient to minimize cost without considering the probability of events. Thus, there is the conditional probability  $P(j | i)$  of application  $j$  selecting equipment type  $i$  in equation (2).

$$PV(i, j) = P(j | i) \left[ C_i + \frac{C_v}{(1+r)^{t_j}} \right] = P(j | i) \left[ C_i + \frac{c_{ij} * t_j}{(1+r)^{t_j}} \right] \quad (2)$$

Subject to:

$$\sum_{j=1}^N X_{ij} \leq M_i \quad \text{for all } i \text{ (cannot assign more equipment than is available) [3]} \quad (3)$$

With respect to constraint (3), it is possible that a given equipment type  $i$  will be assigned to more than one application because of cost considerations.

$$\sum_{j=1}^N t_j \leq (T_j - -TU_j - -TM_j) \quad \text{(time availability constraint per application) [3]} \quad (4)$$

Constraint (4) reflects the fact that we must account for down time due to unreliable operation and the need for maintenance.

$$f_{kj} \geq f_{ki} \quad \text{for all } k \text{ and } i \quad (5)$$

Constraint (5) deals with the requirement that all equipment effectiveness factors must satisfy application  $j$  required effectiveness factors.

## III. Equipment Selection Factors and Cost-Effectiveness

In addition to consideration of cost, we must evaluate the effectiveness of the various equipment factors. For example, if we were selecting computers, we might be interested in clock speed, RAM size, disk size, and auxiliary storage devices. Let us develop a weighted effectiveness metric as follows:

$$E(i, k) = \sum_{i=1}^N \sum_{k=1}^K w_k * F_{ik} \quad \text{for all } i \quad (6)$$

where  $w_k$  is the weight of factor  $k$  and  $F_{ik}$  the *normalized* value of factor  $f_{ik}$ ;  $F_k$  is given in equation (7). The factors are normalized because the factor units (e.g., MHZ, MB) only have meaning for a given factor. Normalization

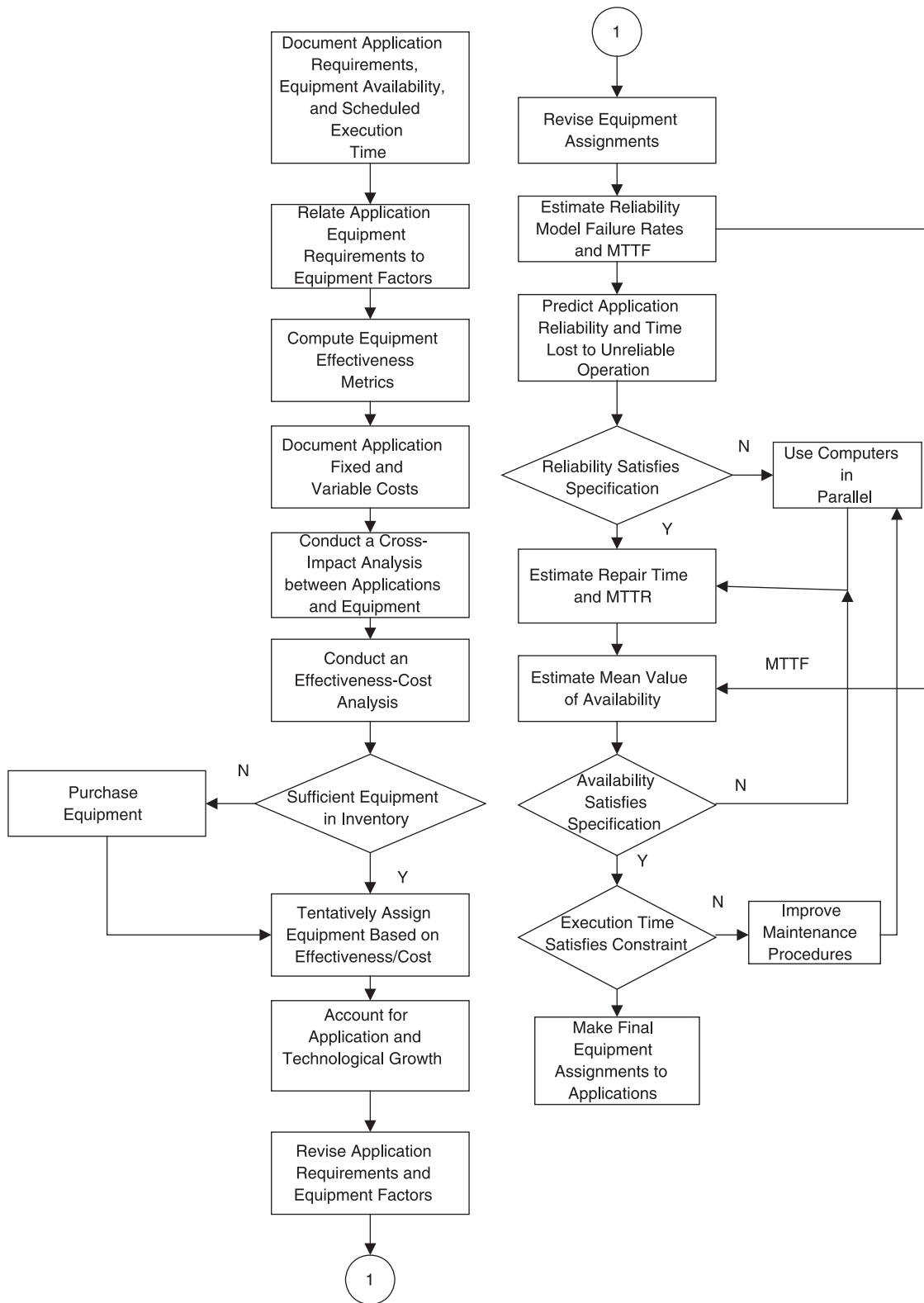


Fig. 1 Process flow diagram of equipment assignment model.

provides a uniform measurement.

$$F_{ki} = \frac{f_{ki}}{\sum_{k=1, i=1}^{K, N} f_{ki}} \quad (7)$$

Then, effectiveness and cost can be integrated in equation (8) as the EC ratio:

$$EC(i, j, k) = E(i, k)/PV(i, j) \quad (8)$$

Our overall objective is to minimize  $EC(i, j, k)$ .

#### IV. Equipment Specifications and Cost Matrices

The first thing to do in equipment assignment is to document application equipment requirements along with equipment availability and scheduled execution time in Table 1. For equipment to be assigned to applications in an intelligent manner, there must be a matrix of properties desired by the customer and vendor specifications and costs that the customer can use to aid selection. We use two matrices: Table 2 to specify application requirements and equipment factors and Table 3 to document vendor specifications and effectiveness metrics. The data in Table 3 is adapted from a well-known vendor’s specifications. Table 4 documents variable costs and fixed vendor costs by equipment types and application. Except for vendor specifications and fixed costs, the data in these tables are assumed in order to illustrate the models.

#### V. Cross-Impact Analysis

Cross-impact analysis is a probabilistic assessment of the relationship between events [7]. For example, we can assess the impact of the event  $e_i$  of selecting equipment type  $i$  on application  $j$  (event  $e_j$ ). This assessment is made by considering the following probabilities:

- $P(i)$  probability of  $e_i$ ;
- $P(j)$  probability of  $e_j$ ;
- $P(j | i)$  probability of  $e_j$  given  $e_i$ .

**Table 1 Application  $j$  equipment requirements, equipment availability  $i$ , and scheduled execution time**

Equipment type $i$ availability	Web service $j = 1$	Safety critical $j = 2$	Home PC $j = 3$
$M_1 =$ three processors	$X_{11} =$ one processor	$X_{12} =$ two processors	$X_{13} =$ one processor
$M_2 =$ three RAMs	$X_{21} =$ one RAM	$X_{22} =$ two RAMs	$X_{23} =$ one RAM
$M_3 =$ three disks	$X_{31} =$ two disks	$X_{32} =$ two disks	$X_{33} =$ one disk
$M_4 =$ three auxiliary cache processors	$X_{41} =$ one auxiliary cache processor	$X_{42} =$ two auxiliary cache processors	$X_{43} =$ one auxiliary cache processor
Scheduled execution time $t_j$ on application $j$ per session or mission (h)	6	20	4
Total application execution time (h)	6 + 20 + 4 = 30		

**Table 2 Application  $j$  equipment requirements and equipment factors  $k$**

Equipment factor $k$	Web service $j = 1$	Safety critical $j = 2$	Home PC $j = 3$
Clock speed $k = 1$	$f_{11} =$ 0.500 GHz	$f_{12} =$ 2.600 GHz	$f_{13} =$ 0.166 GHz
RAM speed for 4 GB $k = 2$	$f_{21} =$ 256 MHz	$f_{22} =$ 512 MHz	$f_{23} =$ 128 MHz
Disk size $k = 3$	$f_{31} =$ 400 GB	$f_{32} =$ 100 GB	$f_{33} =$ 300 GB
Processor cache size $k = 4$	$f_{41} =$ 4 MB	$f_{42} =$ 2 MB	$f_{43} =$ 1 MB

Table 3 Equipment specifications and effectiveness metrics

Equipment type $i$	Clock speed $f_k = 1$ (GHz)	Clock speed $f_k = 1$ normalized	RAM speed for 4 GB		Disk size $f_k = 3$		Processor cache size $f_k = 4$	
			$f_k = 2$ (MHz)	$f_k = 2$ normalized	(GB)	normalized	$f_k = 4$ (MB)	normalized
$i = 1$	1.86	0.1562	533	0.2103	1000	0.2105	2	0.1111
$i = 2$	2.66	0.2233	667	0.2632	500	0.1053	4	0.2222
$i = 3$	3.66	0.3073	667	0.2632	500	0.1053	4	0.2222
$i = 4$	3.73	0.3132	667	0.2632	2750	0.5789	8	0.4444
Equipment factor weight $w_k$		0.5		0.2		0.3		0.1
$i = 1$ Effectiveness Metric	$0.5 * 0.1562 + 0.2 * 0.2103 + 0.3 * 0.2105 + 0.1 * 0.1111 = 0.1944$							
$i = 2$ $E(i) = \sum_{k=1}^K w_k * f_k$ for all $i$	$0.5 * 0.2233 + 0.2 * 0.2632 + 0.3 * 0.1053 + 0.1 * 0.2222 = 0.2181$							
$i = 3$	$0.5 * 0.3073 + 0.2 * 0.2632 + 0.3 * 0.1053 + 0.1 * 0.2222 = 0.2601$							
$i = 4$	$0.5 * 0.3132 + 0.2 * 0.2632 + 0.3 * 0.5789 + 0.1 * 0.4444 = 0.4274$							

**Table 4 Variable costs  $c_{ij}$  (per hour) fixed costs  $C_i$ , and scheduled execution time**

Application $j$	Equipment type $i$				Scheduled execution time used on application $j$ $t_j$ (h)
	$i = 1$	$i = 2$	$i = 3$	$i = 4$	
Web service $j = 1$	200	250	300	500	6
Safety critical $j = 2$	300	350	400	600	20
Home PC $j = 3$	100	150	200	400	4
Fixed cost $C_i$	369	999	1199	5089	

This relationship is expressed as:

$$P(j | i) = \frac{P(i | j) * P(j)}{P(i)} \quad (9)$$

$$P(i) = 1/M_i \quad (\text{inversely related to number of equipments of type } i) \quad (10)$$

$$P(j) = 1/N \quad (\text{inversely related to number of applications}) \quad (11)$$

$$P(i | j) = 1/C_i \quad (\text{inversely related to fixed cost of equipment type } i) \quad (12)$$

Then using equations (10)–(12), equation (9) becomes:

$$P(j | i) = [(1/C_i) * (1/N)] * M_i \quad (13)$$

Table 5 shows the computation of  $P(j | i)$  that will be applied to the PV cost function from equation (2).

## VI. Effectiveness-Cost Analysis

The first factor in EC analysis is to determine whether the equipment availability constraint in equation (3) has been satisfied. We see in Table 1 that this is not the case because the safety critical application requirements exceed the availability. Therefore, the laboratory must purchase additional equipment.

The second factor is to examine the cost-effectiveness of equipment assignments. Figure 2 shows that the most cost-effective solution is to assign equipment type 1 to all applications. However, as shown in Tables 2 and 3 by the bolded quantities, equipment type 1 does not completely satisfy the specifications. Therefore, an alternative solution must be found. By examining Fig. 2 we see that, after excluding equipment type 1, neither equipment type 2 nor 3 have maximum EC ratios for all applications. However, we see from Table 4 that equipment type 2 has the lower fixed cost. Therefore, at this stage in the analysis, we tentatively select equipment type 2.

## VII. Account for Application and Technological Growth

Up to this point nothing has been said about application growth. Applications do not remain static. They grow at an exponential rate as users demand new features and functionality. The justification for the exponential function is based on the growth of the US Internet backbone, as shown in Fig. 3 [8]. Since all applications use the backbone in some fashion, it is appropriate to use an exponential rate of growth for clock speed demand that is a primary determinant of application execution time. Note that the fitted exponential function in Fig. 3, based on [8], has a high  $R^2 = 0.97$ . This demand will be reflected in increases in requirement effectiveness factors  $f_{kj}(n)$ , as a function of the number of years of planned operation by the laboratory that is assumed to be the same for each application. Exponential growth is expressed as follows:

$$f_{kj}(n) = f_{kj} * e^n \quad (14)$$

Now RAM speed, disk size, and cache size demand would grow at a slower, linear rate, expressed as follows:

$$f_{kj}(n) = f_{kj} * n \quad (15)$$

**Table 5 Equipment and application probabilities**

Application $j$	$P(j) = 1/N$	Equipment Type $i$			
		1	2	3	4
		$P(i) = 1/M_i$			
Web service $j = 1$	0.333	$1/369 = 0.0027$	$1/999 = 0.0010$	$1/1199 = 0.0008$	$1/5089 = 0.0002$
Safety critical $j = 2$	0.333	$P(j i) = \frac{P(i j) * P(j)}{P(i)}$	$(0.0010 * 0.333)/0.250 = 0.0013$	$(0.0008 * 0.333)/0.250 = 0.0011$	$(0.0002 * 0.333)/0.250 = 0.0003$
Home computer $j = 3$	0.333	$0.250 = 0.0036$	$0.250 = 0.0013$	$0.250 = 0.0011$	$0.250 = 0.0003$



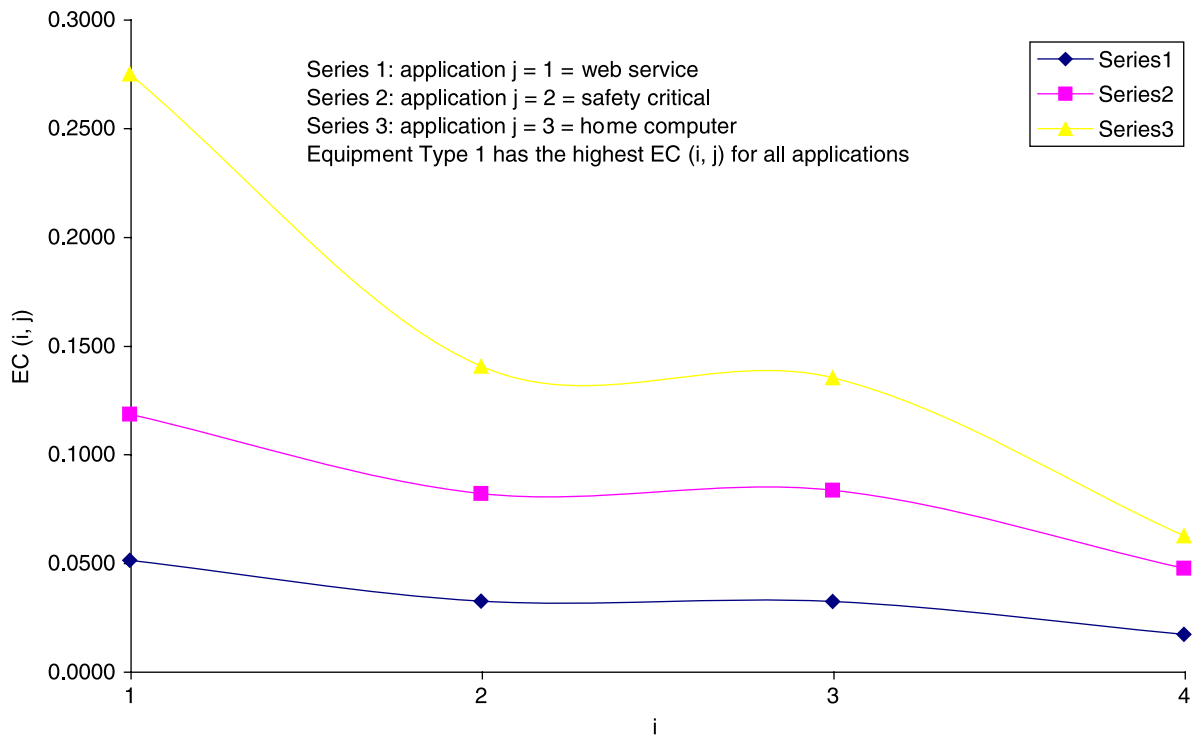


Fig. 2 Effectiveness-cost ratio  $EC(i, j)$  vs equipment model 1.

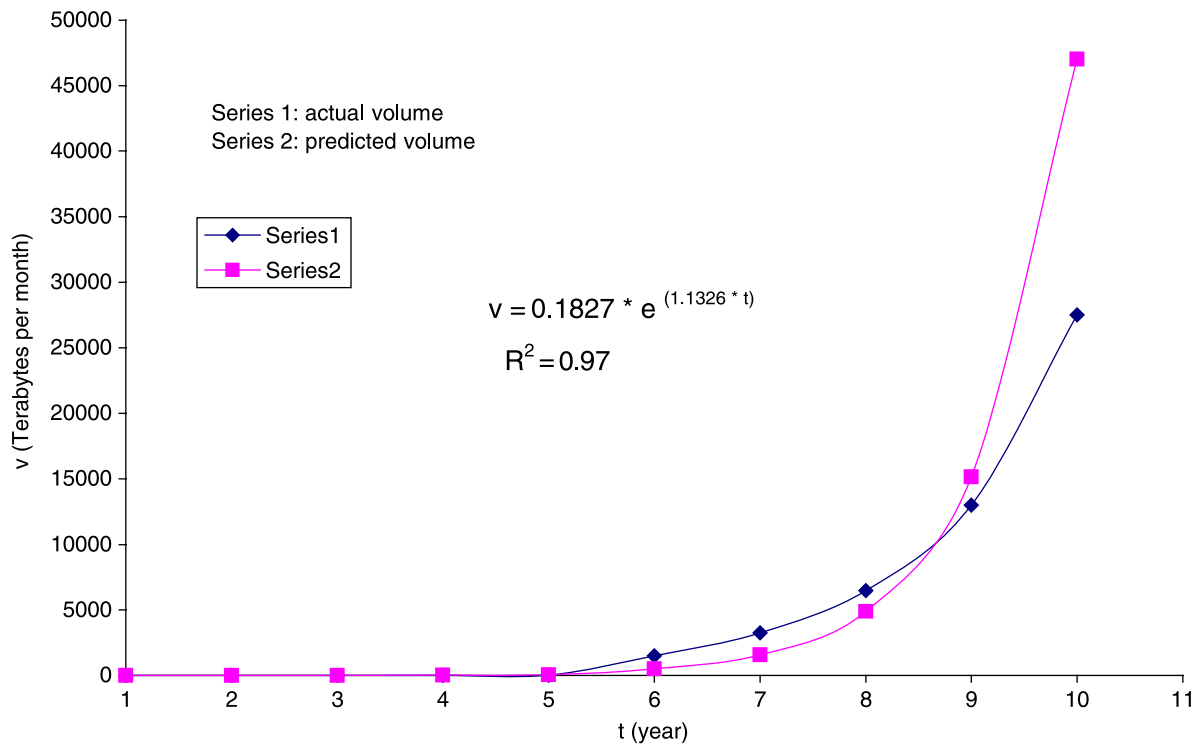


Fig. 3 US internet backbone growth volume  $v$  vs time  $t$ : 1990–2000.

After forecasting application growth in equations (14) and (15) for each effectiveness factor and application, we match the forecasts against the expected growth in vendor effectiveness factors, as governed by Moore's Law.

Moore's Law describes an important trend in the history of computer hardware: the number of transistors that can be inexpensively placed on an integrated circuit is increasing exponentially, doubling approximately every two years. The observation was first made by Intel co-founder Gordon E. Moore. The trend has continued for more than half a century and is not expected to stop for at least a decade and perhaps much longer. Almost every measure of the capabilities of digital electronic devices is linked to Moore's Law: processing speed, memory capacity, even the resolution of LCD screens and digital cameras. All of these are improving at (roughly) exponential rates as well<sup>†</sup>.

Thus Moore's Law, with respect to effectiveness factors, can be expressed as:

$$f_{ki}(n) = (2f_{ki})^{(n/2)} \quad (16)$$

We note that disk size being partly a function of mechanical moving parts may not exhibit the rapid growth of electronic devices that is predicted by Moore's Law.

## VIII. Results of Application and Technological Growth Analysis

### A. Clock Speed

Now, for each application, we determine which of the equipment types satisfies the application. It is likely that there will not be a consistent choice of equipment type for a given application for all effectiveness factors. In that case, it would be necessary to make EC trade-offs to determine the "best" choice.

Due to the large number of figures that are involved, we show only selected examples. The first result, shown in Fig. 4, indicates that equipment types 3 and 4 satisfy the web service application. Since from Fig. 2, equipment

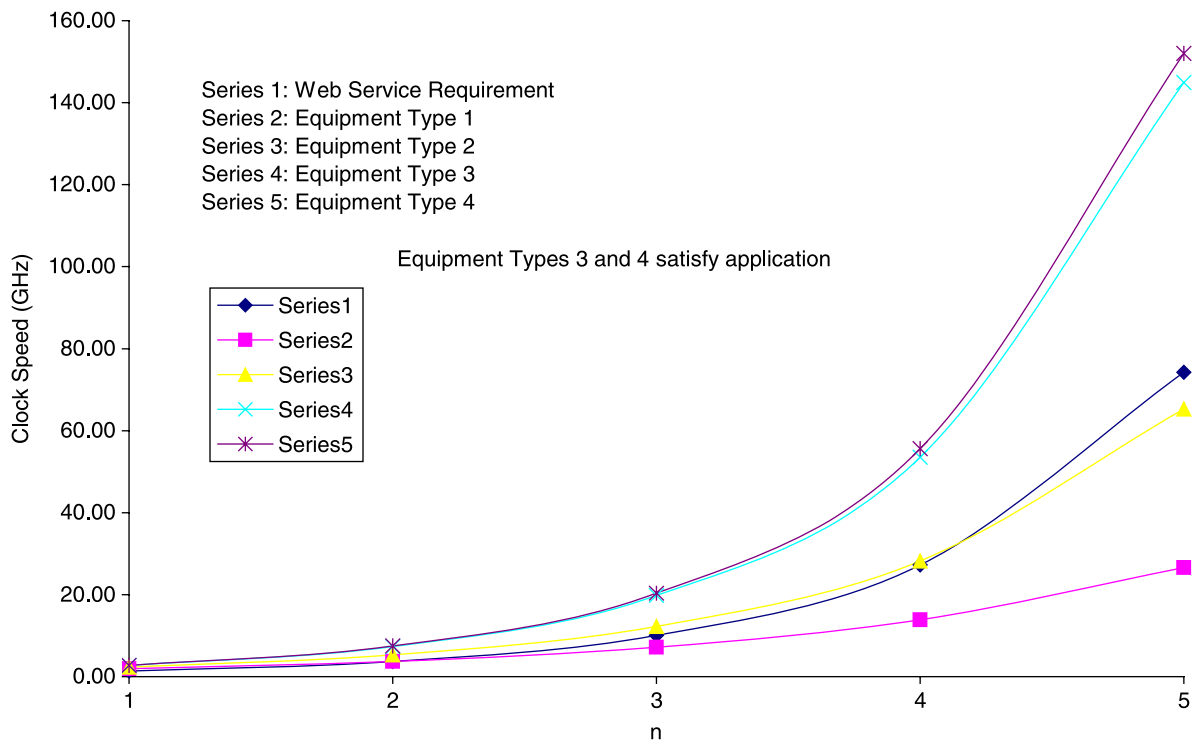


Fig. 4 Clock speed vs year  $n$ .

<sup>†</sup> "Moore's law" [http://en.wikipedia.org/wiki/Moore's\\_law](http://en.wikipedia.org/wiki/Moore's_law) [retrieved 3 June 2009]

type 3 has the higher EC ratio, we would choose this equipment type for this application. The same result was obtained for the safety critical application (not shown). In the case of the home computer application, all equipment types satisfy the application requirements. Again, referring to Fig. 2, we would choose the most effective-cost alternative equipment type 1 for the home computer application.

### B. RAM Speed

Figure 5 shows that for the safety critical application, the requirement for RAM speed cannot be satisfied beyond the third year of operation by equipment types 2, 3, and 4. In addition, the requirement cannot be met beyond year 1 by equipment type 1 (not shown). The recourse would be to upgrade equipment type 3, the highest feasible EC ratio alternative (see Fig. 2) at year 3, to a RAM speed of 2.56 GHz, the speed required at year 5. In contrast, the RAM speed requirement is satisfied for the other two applications over a period of five years by equipment types 2, 3, and 4. Again, consulting Fig. 2, we see that type 2 has the highest EC of the feasible alternatives, and should be assigned.

### C. Disk Size

In Fig. 6 we see that equipment type 1, the most cost-effective model, easily satisfies the disk size requirements of all applications. Therefore, it is the obvious choice based on disk size.

### D. Cache Size

From Fig. 7 it is seen that equipment type 2 easily satisfies all application requirements. Equipment type 1 did not achieve this distinction (not shown) because it did not satisfy all requirements. Thus, equipment type 2 is the choice for all applications.

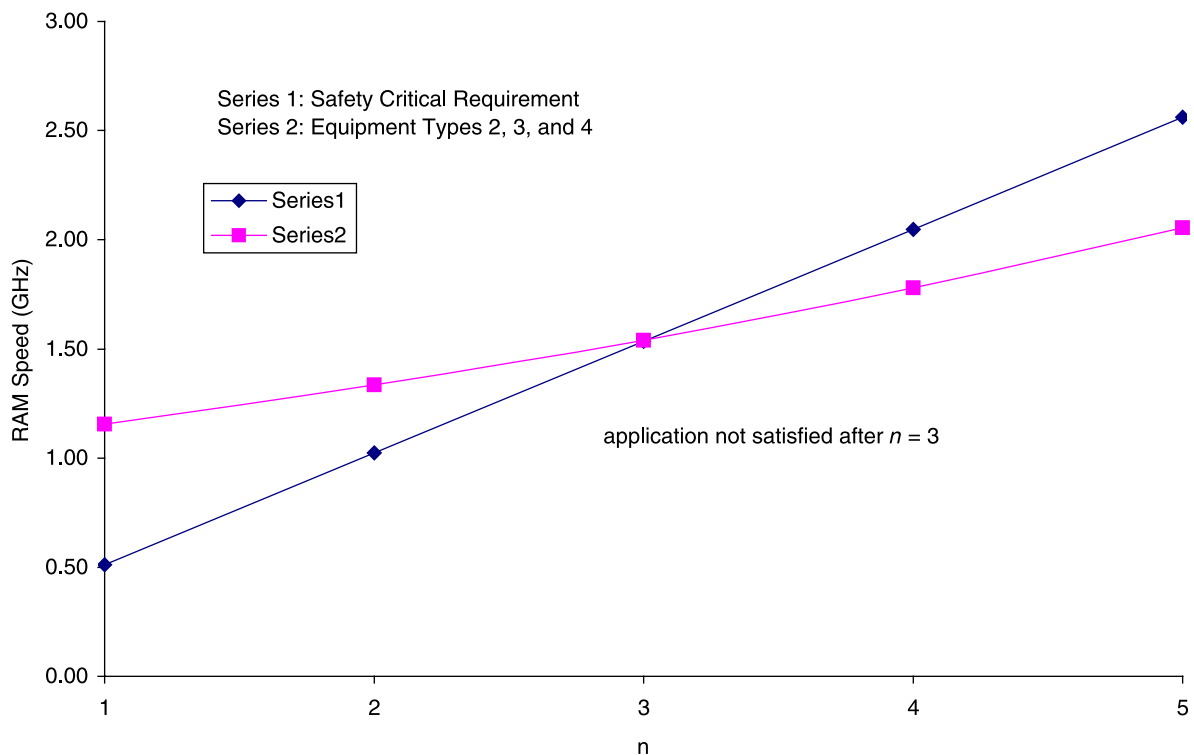


Fig. 5 RAM speed for 4 GB RAM vs year  $n$ .

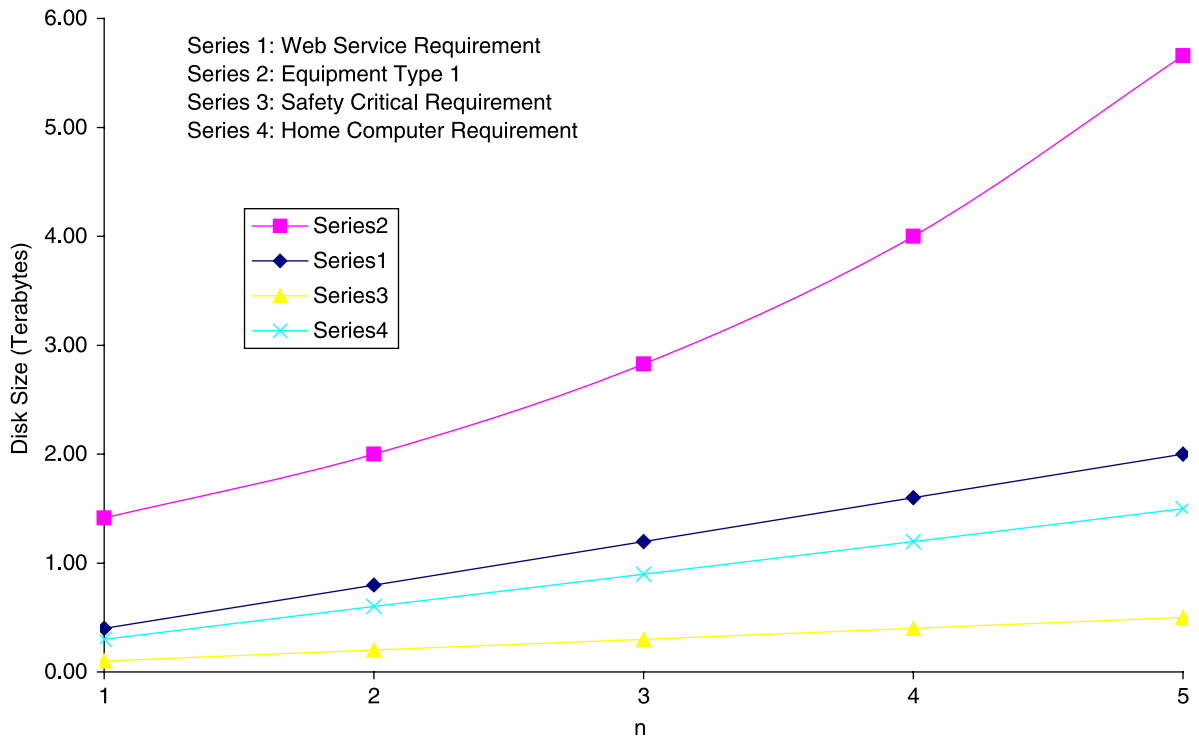


Fig. 6 Disk size vs year *n*.

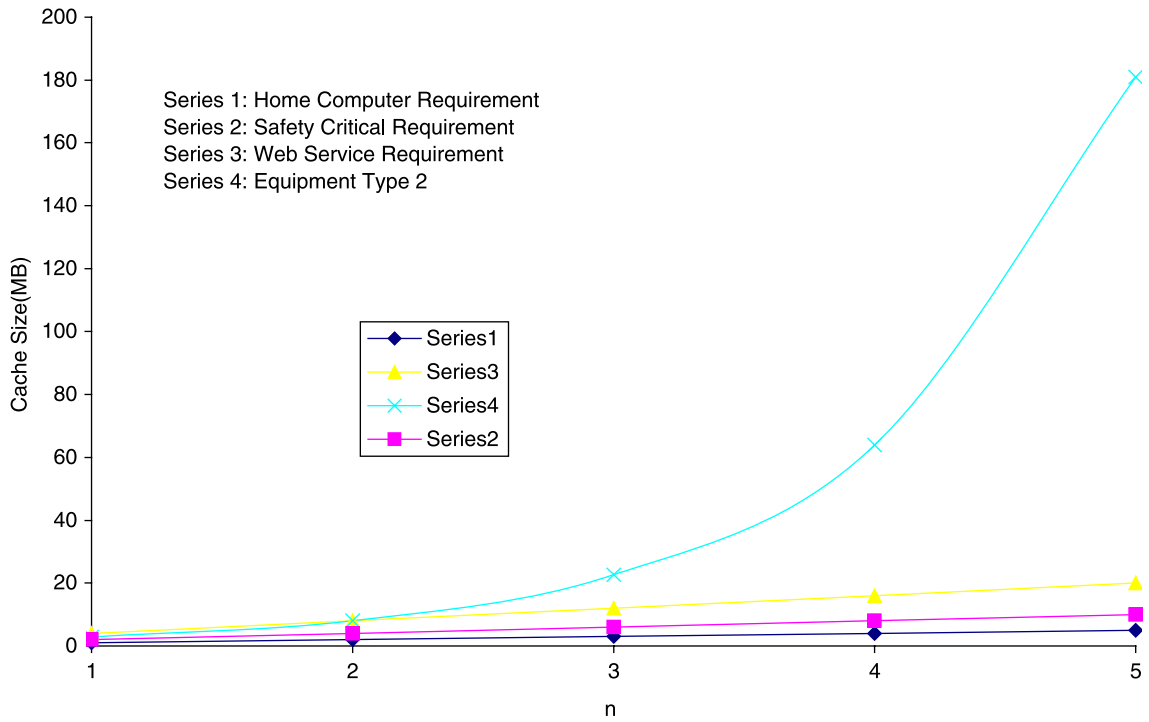


Fig. 7 Cache size vs year *n*.

**Table 6 Assignment of effectiveness factors  $k$  to applications**

Equipment type $i$	Clock speed $k = 1$	RAM speed $k = 2$	Disk size $k = 3$	Cache size $k = 4$
1	Home computer		Web service, safety critical, home computer	
2		Web service, home computer		Web service, safety critical, home computer
3	Web service, safety critical	Safety critical: upgrade to 2.6 GHz at year 3		
4	No assignment	No assignment	No assignment	No assignment

### E. Summary

It is appropriate at this point to summarize the results thus far to identify the best assignments of equipment effectiveness factors to applications in Table 6. We see that thus far, equipment type 4 is out of the running and that the other equipment types are still in contention.

## IX. Reliability Model

Thus far we have not addressed reliability. The implicit assumption has been that the equipment operates with 100% reliability. From our experience with personal computers we know this is not the case! In classical hardware reliability theory the failure rate is assumed to be constant over operating time [9]. This assumption is appropriate for solid state devices where the physics of the devices is well known. Again, from experience we know this is not the case with computers. Over time circuit boards can degrade and mechanical parts can wear out. Therefore, failure rate must be modeled as a function of time.

Thus the reliability of an application's hardware, operating for a time  $t_j$ , is

$$R(t_j) = e^{-\lambda(t_j)t_j} \quad (17)$$

where the failure rate  $\lambda(t_j)$  is a function of  $t_j$  and not constant. We obtained some empirical failure rate data as follows.

Gartner [10] says that systems bought in 2003 and 2004 had a seven percent failure rate in the first year, rising to 15 percent by the fourth year of usage. The first year failure rate fell to five percent ( $\lambda_1 = 0.05$  per unit time) for systems bought in 2005 and 2006. It is projecting a failure rate of 12 percent (0.12 per unit time) in year 4. This computes to a mean of  $r = 0.0233$  failures per unit time. For computational convenience, this information can be translated into the failure rate function in equation (18) by putting the failure rates on an  $T_j = 100$  h application and equipment operational schedule.

$$\lambda(t_j) = \lambda_1 + r(t_j - 1) \quad (18)$$

or  $\lambda(t_j) = 0.0005 + (0.000233 * (t_j - 1))$ .

Then using equations (17) and (18), the reliability becomes:

$$R(t_j) = e^{-[\lambda_1 + r(t_j - 1)] * t_j} \quad (19)$$

We also want to estimate the mean time to failure (MTTF) so that it can be used later in availability and maintainability calculations. MTTF is defined as the reciprocal of the failure rate [9]. In this model, it is not a constant, as in classical hardware reliability theory. Rather, it is a function of application execution time as follows:

$$\text{MTTF}(t_j) = \frac{1}{\lambda(t_j)} = \frac{1}{\lambda_1 + r(t_j - 1)} \quad (20)$$

Finally, in order to determine whether the time availability constraint of equation (4) is met, we need to predict the time an application is inoperable due to unreliable operation in equation (21).

$$TU_j = (1 - R(t_j)) * T_j \quad (21)$$

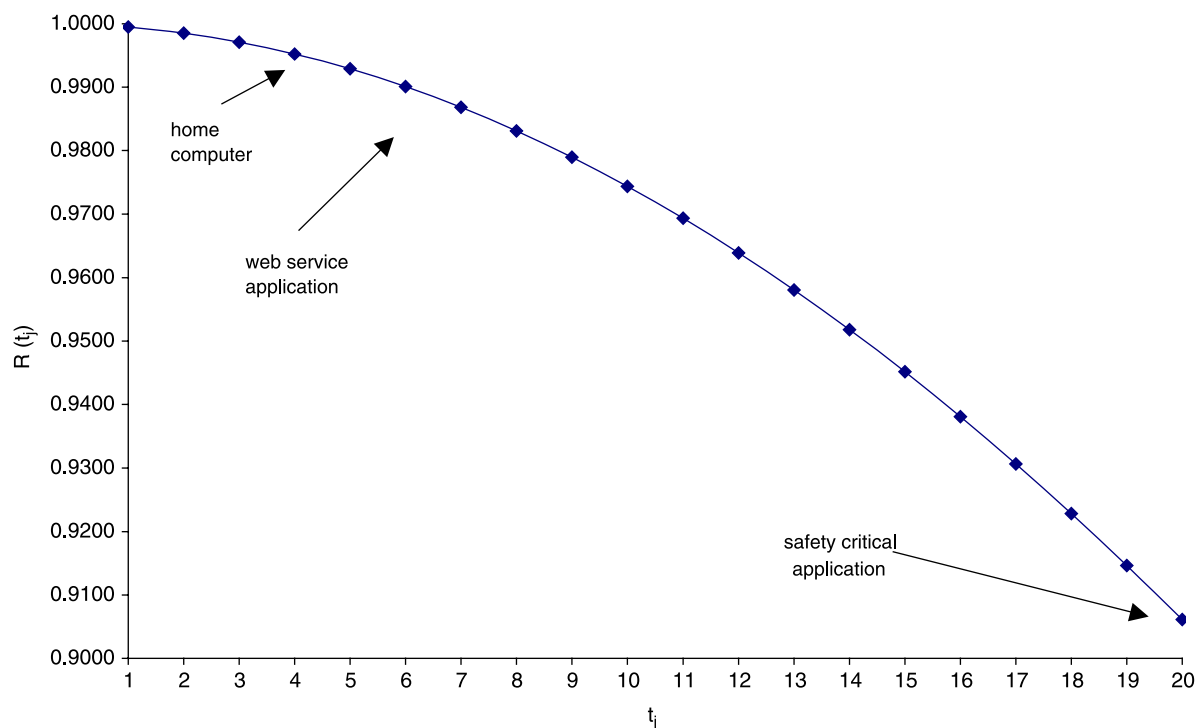


Fig. 8 Reliability  $R(t_j)$  of applications vs time of operation of applications  $t_j$ .

### A. Results of Reliability Analysis

Figure 8 shows that the reliability for the safety critical application is too low; it should be at least 0.95. The problem is the relative high mean failure rate. A resolution of the problem is to use redundancy (i.e., computers operating in parallel [9]). Thus, we will experiment with  $n$  computers in parallel. We will account for the additional cost and the change in cost-effectiveness. The reliability of  $n$  computers in parallel with equal reliabilities  $R(t_j)$  is computed by

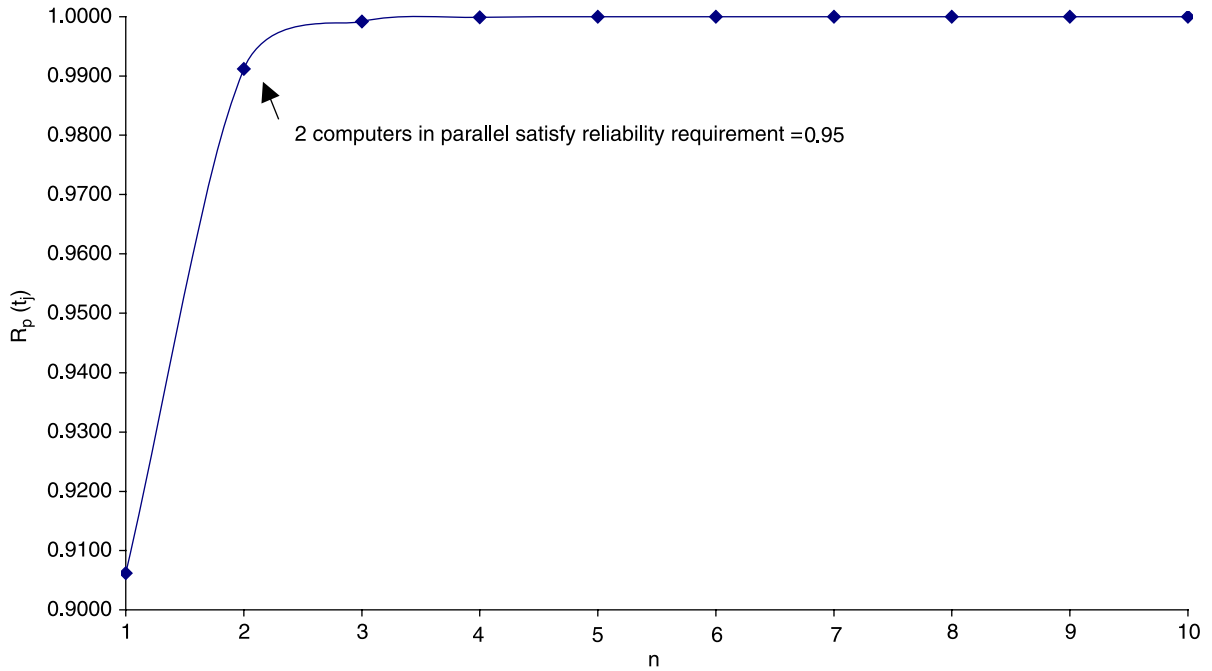
$$R_p(t_j) = 1 - (1 - R(t_j))^n \quad (22)$$

From Fig. 9 we see that the reliability problem of the safety critical application can be solved by using two computers in parallel. Now we need to see how this solution affects cost-effectiveness. The safety critical application gains a 4.88% increase in reliability from 0.9062 to 0.9500 by using two computers in parallel. This gain comes with a penalty of 33.41% decrease in EC ratio from 0.0838 to 0.0058, using equipment type 3. Obviously, this is a significant penalty but, given that this is a safety critical application, it is mandatory that the reliability goal be met.

## X. Maintainability and Availability Model

### A. Maintainability

Of course it is also important to account for the down time of equipment attributed to maintenance, which, in turn, is the result of unreliable operation, and adversely affects the availability of the equipment for running the applications. We assume that maintenance time is exponentially distributed, meaning there is a higher probability of short maintenance times versus long ones. This seems reasonable because most repair tasks would be relatively minor and short-lived (e.g., replacing a monitor) compared to major repair events (e.g., replacing a burned out mother board). We have done an extensive search of the Internet looking for empirical data on computer repair rates, but we have come up empty-handed; therefore, we are forced to improvise with what we believe is a reasonable approach. We assume that the repair rate is *proportional* to the failure rate of equipment because it is natural that there would be a maintenance action in response to a failure. However, the repair rate is not identical to the failure rate. Because the



**Fig. 9 Reliability of safety critical application  $R_p(t_j)$  with  $n$  computers in parallel vs  $n$ .**

repair rate represents maintenance actions like using test equipment, trying spare components, and running diagnostic routines, the repair rate is higher than the failure rate. We represent this relationship by the *acceleration factor*  $m_a$  that is applied to the failure rate in the exponential density function in equation (23). To illustrate the computations, we use  $m_a = 1, 2, 3, 4,$  and  $5$ .

$$M_j(TM_j) = \lambda(t_j) * m_a e^{(-\lambda(t_j) * m_a * TM_j)} \quad (23)$$

where  $M_j(TM_j)$  is the maintainability, the probability that a repair will require a time  $TM_j$ . Now, we are most interested in estimating the repair times for given values of maintainability. Thus, we can solve equation (23) for  $TM_j$ .

$$TM_j = \frac{\log(\lambda(t_j) * m_a / M(TM_j))}{\lambda(t_j) * m_a} \quad (24)$$

In order to not bias the designation of given values of  $M_j(TM_j)$  in equation (24), this quantity is randomized by a uniformly distributed number  $0 \leq M_j(TM_j) < 1.0$ .

Because the repair rate is *proportional* to the failure rate, the mean time to repair is computed as follows:

$$MTTR(t_j) = \frac{1}{\lambda_j * m_a} \quad (25)$$

## B. Availability

We can now combine the reliability and maintainability models to produce the availability model. The *mean* value of availability is defined as the probability that a system is available when needed [9]. From this definition we can formulate the mean availability by using equations (20) and (25) as follows:

$$A(t_j) = \frac{MTTF(t_j)}{MTTF(t_j) + MTTR(t_j)} \quad (26)$$

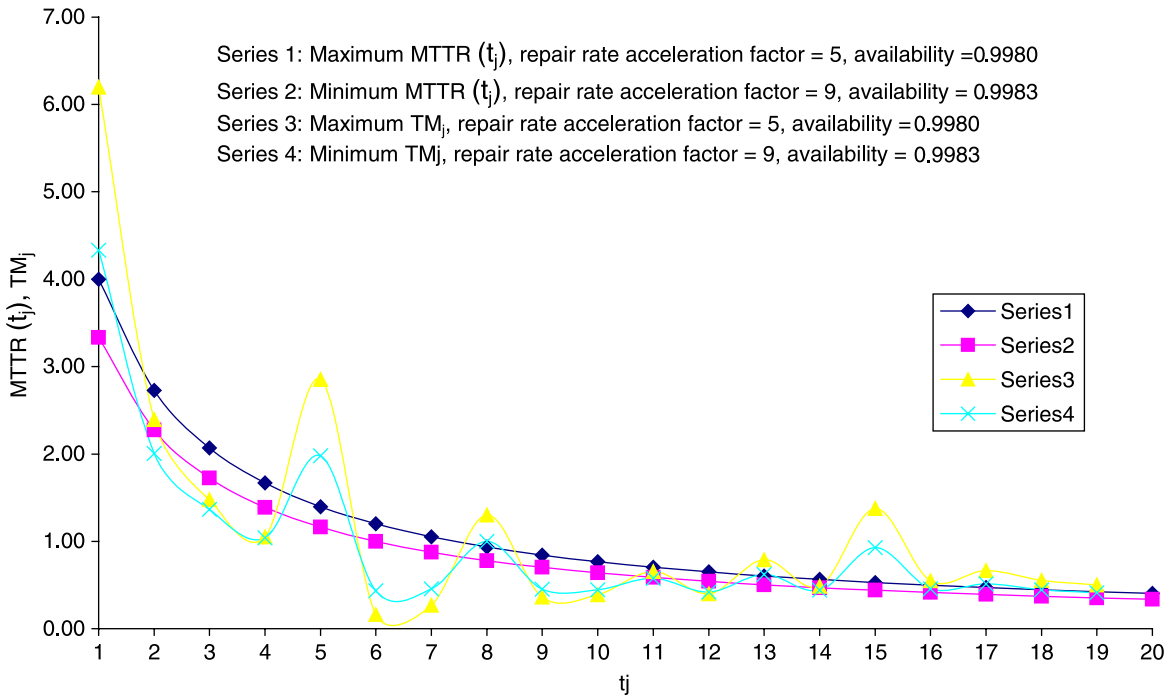


Fig. 10 Mean time of repair (MTTR( $t_j$ )) and time to repair ( $TM_j$ ) vs execution time of application  $t_j$ .

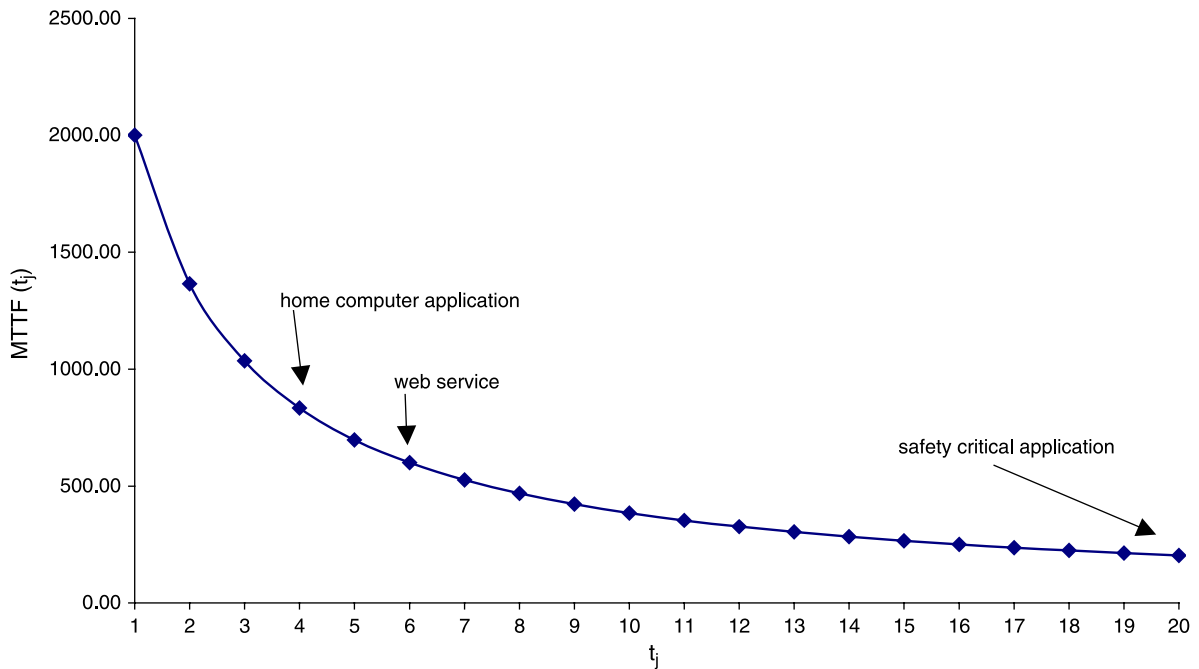


Fig. 11 Mean time to failure (MTTF( $t_j$ )) vs application execution time ( $t_j$ ).



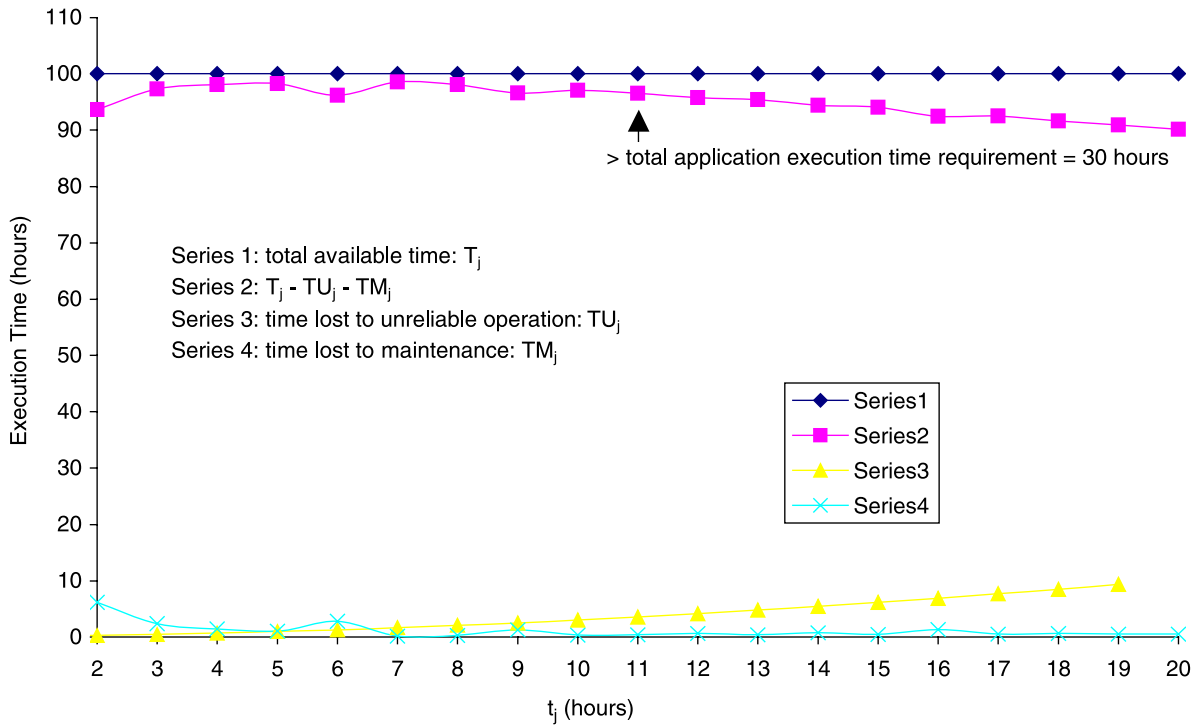


Fig. 12 Required application execution times vs scheduled application time  $t_j$ .

**C. Results of Maintainability and Availability Analysis**

Figure 10 captures the maintainability and availability situation on an integrated basis. That is, assuming an availability specification of 0.99, it is demonstrated that the applications would have this degree of availability. We bound the solution using the minimum and maximum failure rate acceleration factors described previously. We also look at the other component of availability in equation (26),  $MTTF(t_j)$ . This is plotted in Fig. 11, where we see that for each application, indexed by its execution time, the  $MTTF(t_j)$  is significantly greater than the execution time. This result contributes to satisfying the availability requirement. If availability did not meet the specification, reliability would need to be increased to provide an even greater MTTF. Since failure rates are not likely to decrease for the computers that are in use, a solution would be to employ parallel computers on *all* applications.

**D. Results of Execution Time Scheduling Analysis**

Once the down times attributed to unreliable operation and maintenance have been accounted for, we can determine whether the application execution time constraint expressed in equation (4) has been satisfied. This we do in Fig. 12, where it is seen that after accounting for all downtime, the schedule is easily satisfied. If the constraint was not met, the solution would be to improve maintenance procedures in order to reduce repair time and increase reliability by again employing parallel computers.

In summary, remembering to use a parallel computer configuration for the safety critical application, there is no problem with respect to reliability, maintainability, availability, and operational schedule for any of the applications. Therefore, the assignment of equipments to applications remains as indicated in Table 6.

**XI. Conclusions**

Based on using our equipment selection models, we conclude that it is helpful for an engineer to step through the modeling process because doing so is valuable to understanding the trade-offs among cost, effectiveness, reliability, maintainability, and availability. Furthermore, based on the examples that were presented, we conclude that the modeling process must be a step-by-step approach, with possible backtracking over previous solutions, because

there can be occasional surprises in the quantitative results obtained (e.g., parallel computers may be required to satisfy requirements). In addition, the engineer may find it necessary to use hypothetical data in some parts of the model because data such as repair rates may not be available.

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